Robinson Crusoe model

- 1 consumer & 1 producer & 2 goods & 1 factor:
  - two price-taking economic agents
  - two goods: the labor (or leisure x_1) of the consumer and a consumption good x_2 produced by the firm
  - the consumer has continuous, convex, and strongly monotone preferences
  - the consumer has an endowment of $\bar{\boldsymbol{L}}$ units of leisure and no endowment of the consumption good
  - the firm uses labor l to produce the consumption good
  - the production function $f(l)$ is increasing and strictly concave
  - the firm is owned by the consumer

Competitive allocation

- What is the competitive equilibrium allocation for x_1 and x_2?

$$\begin{cases} \max\limits_{l \geq 0} pf(l) - wl \\ \max\limits_{x_1, x_2} u(x_1, x_2) \end{cases}$$ such that $px_2 \leq w(-x_1) + \pi(p,w)$

p – the price of output

w – the price of labor

l(p,w) – the firm's optimal labor demand

q(p,w) – the firm's output (consumption good supply)

$\pi(p,w)$ – the firm's profit

$u(x_1,x_2)$ – utility function

Excess demand for labor – the firm wants more labor than the consumer is willing to supply. (**gr. 10**)

Solution (**gr. 11**)

- The budget line is exactly the isoprofit line. A Walrasian (competitive) equilibrium in this economy involves a price vector *(p*,w*)* at which the consumption and labor markets clear:

$$\begin{cases} \boldsymbol{x_2(p^*,w^*) = q(p^*,w^*)} \\ \boldsymbol{l(p^*,w^*) = \bar{L} - x_1(p^*,w^*)} \end{cases}$$

- There is a unique Pareto optimal consumption vector (and unique equilibrium).

2 x 2 production model

- 2 firms, indexed *j*, each produces a consumer good q_j using K primary good, indexed *l*
- consumers are endowed with the primary goods $\bar{\boldsymbol{l}}_{\boldsymbol{k}}$, but do not demand them (do not consume them).
- factors are immobile and must be used for production within the country. They are traded in the national markets at strictly positive prices *w*.
- the production function $f_j(l_j)$ is concave, strictly increasing, differentiable, and homogeneous of degree one
- the cost function $c_j(w, q_j)$ exists and is differentiable
- there are no intermediate goods
- output is sold in world markets
- output levels q_j are strictly positive (no full specialization)
- output prices p_j are fixed (small open economy, i.e. one of the consumers is abroad)

Equilibrium in the factor markets

- Given the output prices and input prices (***p*, w**), each firm maximizes

$$\max_{\boldsymbol{l}_j \geq 0} \boldsymbol{p}_j \boldsymbol{f}_j(\boldsymbol{l}_j) - \boldsymbol{w}\boldsymbol{l}_j$$

- We can derive their demands for inputs (factors) $l_j(\boldsymbol{p}, \boldsymbol{w})$.
- Market clearing requires that 2 conditions are satisfied:
- **(1)** $l_j^* \in l_j(p, w)$ for all $j = 1, \ldots, J$
- **(2)** $\sum_j \boldsymbol{l}_{kj}^* = \bar{\boldsymbol{l}}_k$ for all $k = 1, \ldots, K$

Equilibrium cont.

The 2 conditions can be re-stated as first order conditons:

(1) $$p_j \frac{\partial f_j(l_j^*)}{\partial l_{kj}} = w_k^*$$ for $j = 1, \ldots, J$ and $k = 1, \ldots, K$

(2) $$\sum_j l_{kj}^* = \bar{l}_k$$ for all $k = 1, \ldots, K$

OR as:

(1) $$p_j = \frac{\partial c_j(w^*, q_j^*)}{\partial q_j}$$ for $j = 1, \ldots, J$

(2) $$\sum_j \frac{\partial c_j(w^*, q_j^*)}{\partial w_k} = \bar{l}_k$$ for all $k = 1, \ldots, K$

The above conditions determine the equilibrium output levels are $q_j^* = f_j(l_j^*)$

Properties of equilibria

- Equilibria must be Pareto efficient
- The Pareto set must lie all above or all below or be coincident with the diagonal of the Edgeworth box. (gr. 12)
- Let $a_j(w) = (a_{1j}(w), a_{2j}(w))$ denote the input combination which minimizes the cost of production of good j.
- Def.: The production of good 1 is relatively *more intensive in factor 1* then the production of good 2, if

$$a_{11}(w)/a_{21}(w) > a_{12}(w)/a_{22}(w)) \qquad \text{for all } \boldsymbol{w} \text{ (gr. 13)}$$

Theorems

- As long as economy does not specialize in the production of a single good, the equilibrium factor prices depend only on the technologies of the firms and on the output prices (factor price equalization theorem). The prices of nontradable factors are equalized across nonspecialized countries.
- Stolper-Samuelson Theorem - If p_j increases, then the equilibrium price of the factor more intensively used in the production of good *j* increases, while the price of the other factor decreases.(gr.14)
- Rybczynski Theorem - If the endowment of a factor increases, then the production of the good that uses this factor relatively more intensively increases and the production of the other good decreases. (gr. 15)

  Exercise: Prove that proportional increase in the production of the good that uses the increased factor relatively more intensively is greater than the proportional increase in the endowment of the factor.